Final-state Rescattering Effects on $B_d \to \pi\pi$ Decays and CP Violation

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Abstract

The significance of final-state interactions in $B_d \to \pi\pi$ decays is phenomenologically demonstrated by taking elastic $\pi\pi \rightleftharpoons \pi\pi$ and inelastic $\pi\pi \rightleftharpoons D\bar{D}$ rescattering effects into consideration. We find that the present experimental data on $B_d^0 \to \pi^+\pi^-$ can well be understood in this approach without fine-tuning of the input parameters, and large CP-violating asymmetries are expected to manifest themselves in such charmless rare processes.

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Recently the branching ratio of $B_d^0 \to \pi^+\pi^-$ has been measured independently by CLEO, BABAR and BELLE Collaborations:

$$\mathcal{B}(B_d^0 \to \pi^+\pi^-) = \begin{cases} \left(4.3^{+1.6}_{-1.4} \pm 0.5\right) \times 10^{-6} & (\text{CLEO[1]}) , \\ \left(6.3^{+3.9}_{-3.5} \pm 1.6\right) \times 10^{-6} & (\text{BELLE[2]}) , \\ \left(9.3^{+2.6+1.2}_{-2.3-1.4}\right) \times 10^{-6} & (\text{BABAR[3]}) . \end{cases} \qquad (1)$$

Theoretical predictions for $\mathcal{B}(B_d^0 \to \pi^+\pi^-)$, as those given in Refs. [4, 5, 6, 7] based on the QCD-improved factorization, are in good agreement with the BABAR data but difficult to coincide with the CLEO data. It has to be seen, in the near future, how three measurements will reach full consistency and whether the final experimental result can well be understood in the factorization approach without fine-tuning of the input parameters.

If the present CLEO data are taken seriously, it seems necessary to modify the theoretical prediction for $\mathcal{B}(B_d^0 \to \pi^+\pi^-)$. To do so one naturally speculates that final-state interactions in $B \to \pi\pi$ decays might be significant and ought not to be ignored in the factorization approach. In Refs. [8, 9] the elastic $\pi\pi \rightleftharpoons \pi\pi$ rescattering effects on the branching ratios and CP asymmetries of $B \to \pi\pi$ transitions have been demonstrated to be important. It is also likely that such rare nonleptonic processes are deeply involved in inelastic final-state interactions [10, 11]. However, it remains a big challenge today to handle the inelastic rescattering effects on B decays in a systematic and quantitative way [12].

In this paper we attempt to follow a purely phenomenological approach to demonstrate the influence of inelastic final-state interactions on $B_d \to \pi\pi$ decays and CP violation. The essential argument is that there may *a priori* exist a two-step process $B_d \to D\bar{D} \Longrightarrow \pi\pi$, arising from inelastic $\pi\pi \rightleftharpoons D\bar{D}$ rescattering, in addition to the direct decay mode $B_d \to \pi\pi$. We find that both elastic and inelastic rescattering effects are possible to modify the predictions based on the QCD-improved factorization. More precise measurements of $B \to \pi\pi$ decays are expected to clarify whether final-state interactions in them are really significant or not.

It is well known that the final state $\pi^+\pi^-$ or $\pi^0\pi^0$ of B_d decays consists of both $I = 0$ and $I = 2$ isospin configurations, and D^+D^- or $D^0\bar{D}^0$ consists of both $I = 0$ and $I = 1$ isospin configurations. Under inelastic rescattering the $I = 0$ configuration of $\pi\pi$ can mix with that of $D\bar{D}$, leading to a two-step decay mode $B_d \to D\bar{D} \Longrightarrow \pi\pi$. The final states $\pi^\pm\pi^0$ of $B_u^\pm$ decays, which only have the $I = 2$ isospin configuration, cannot mix with $D\bar{D}$. But $\pi^\pm\pi^0$ are possible to mix with the final states like $\rho^\pm\rho^0$, and $\pi^+\pi^-$ (or $\pi^0\pi^0$) could also mix with the final states such as K^+K^- and $K^0\bar{K}^0$ [11]. For simplicity, we assume that the inelastic final-state interactions of $B_d \to \pi\pi$ decays are dominated by the $I = 0$ channel mixing via $\pi\pi \rightleftharpoons D\bar{D}$ scattering, leaving the $I = 2$ state of $\pi\pi$ unmixed with others. In the assumption made above and in the neglect of small electroweak penguin contributions, the amplitudes of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^0\pi^0$, and $B_u^+ \to \pi^+\pi^0$ decay modes can be written as [13] †:

$$\begin{aligned} A(B_d^0 \to \pi^+\pi^-) &= \sqrt{2}S_2^{\pi\pi}A_2^{\pi\pi} + \sqrt{2}\left(S_0^{\pi\pi}A_0^{\pi\pi} + S_0^{\pi D}A_0^{D\bar{D}}\right) , \\ A(B_d^0 \to \pi^0\pi^0) &= 2S_2^{\pi\pi}A_2^{\pi\pi} - \left(S_0^{\pi\pi}A_0^{\pi\pi} + S_0^{\pi D}A_0^{D\bar{D}}\right) , \\ A(B_u^+ \to \pi^+\pi^0) &= 3A_2^{\pi\pi} , \end{aligned} \qquad (2)$$

where $A_{0,2}^{\pi\pi}$ denote the "bare" isospin amplitudes of $B \to \pi\pi$, $A_0^{D\bar{D}}$ stands for the $I = 0$ isospin amplitude of $B_d \to D\bar{D}$ [14], $S_{0,2}^{\pi\pi}$ and $S_0^{\pi D}$ are the inelastic-rescattering matrix ele-

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†Note that the sign of $A_0^{\pi\pi}$ is here taken to be different from that in Ref. [13]. The present choice will prove convenient when the factorization approximation is applied to the isospin amplitudes.

ments connecting the unitarized isospin amplitudes to the bare ones [11]. Obviously $S_0^{\pi D} = 0$ and $S_2^{\pi\pi} = S_0^{\pi\pi} = 1$ held, if there were no mixture between the $I = 0$ states of $\pi\pi$ and $D\bar{D}$.

The bare isospin amplitudes $A_{0,2}^{\pi\pi}$ and $A_0^{D\bar{D}}$ can be calculated with the help of the effective weak Hamiltonian [15] and the QCD-improved factorization [7]. After a straightforward calculation, we obtain

$$
\begin{aligned}
A_0^{\pi\pi} &= \frac{G_F}{6} \left[V_{ub}^* V_{ud} \left(2a_1 - a_2 + 3a_4^u + 3a_6^u \xi_\pi \right) + V_{cb}^* V_{cd} \left(3a_4^c + 3a_6^c \xi_\pi \right) \right] T_\pi e^{i\delta_0} , \\
A_2^{\pi\pi} &= \frac{G_F}{6} V_{ub}^* V_{ud} (a_1 + a_2) T_\pi e^{i\delta_2} , \\
A_0^{D\bar{D}} &= \frac{G_F}{\sqrt{2}} \left[V_{ub}^* V_{ud} \left(a_4^u + a_6^u \xi_D \right) + V_{cb}^* V_{cd} \left(a_1 + a_4^c + a_6^c \xi_D \right) \right] T_D e^{i\delta_0} ,
\end{aligned}
\tag{3}
$$

in which V_{ub}, V_{ud}, V_{cb}, and V_{cd} are the quark mixing matrix elements; a_1, a_2, $a_4^{u,c}$, and $a_6^{u,c}$ are the QCD coefficients independent of the renormalization scheme [4]; ξ_π and ξ_D are the factorization parameters arising from the transformation of $(V - A)(V + A)$ currents into $(V - A)(V - A)$ ones for the penguin operators Q_5 and Q_6 [15]; δ_0 and δ_2 are strong (isospin) phases, T_π and T_D denote the factorized hadronic matrix elements of $B \to \pi\pi$ and $B \to D\bar{D}$ decays respectively. Under isospin symmetry ξ_π and ξ_D read

$$
\begin{aligned}
\xi_\pi &= \frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)} , \\
\xi_D &= \frac{2m_D^2}{(m_b - m_c)(m_c + m_d)} .
\end{aligned}
\tag{4}
$$

In terms of the relevant decay constants and form factors, one gets

$$
\begin{aligned}
T_\pi &= i f_\pi F_0^{B\pi}(m_\pi^2) \left(m_B^2 - m_\pi^2 \right) , \\
T_D &= i f_D F_0^{BD}(m_D^2) \left(m_B^2 - m_D^2 \right) .
\end{aligned}
\tag{5}
$$

It is worth remarking that the isospin amplitudes $A_{0,2}^{\pi\pi}$ and $A_0^{D\bar{D}}$ have been calculated separately in the factorization approximation, hence the contribution of $A_0^{D\bar{D}}$ to $A(B_d \to \pi\pi)$ need in principle be normalized. Such a treatment is however unnecessary in the approach under discussion, because the normalization factor of $A_0^{D\bar{D}}$ can always be absorbed into the unknown parameter $S_0^{\pi D}$ in Eq. (2). This point will be seen more clearly later on.

Following Eq. (2) one may write down the similar isospin relations for the amplitudes of $\bar{B}_d^0 \to \pi^+\pi^-$, $\bar{B}_d^0 \to \pi^0\pi^0$, and $B_u^- \to \pi^-\pi^0$ decay modes, whose bare isospin amplitudes $\bar{A}_{0,2}^{\pi\pi}$ and $\bar{A}_0^{D\bar{D}}$ can directly be read off from $A_{0,2}^{\pi\pi}$ and $A_0^{D\bar{D}}$ in Eq. (3) with the replacements $V_{ub}^* V_{ud} \Longrightarrow V_{ub} V_{ud}^*$ and $V_{cb}^* V_{cd} \Longrightarrow V_{cb} V_{cd}^*$. Of course $|\bar{A}_2^{\pi\pi}/A_2^{\pi\pi}| = 1$ holds in the approximation of neglecting the electroweak penguin effect.

Let us give a brief retrospection of the conventional calculations of $B \to \pi\pi$ decays, in which final-state interactions are considered only at the quark level [4, 5, 6, 7]. Taking $\delta_0 = \delta_2 = 0$, $S_0^{\pi D} = 0$, and $S_0^{\pi\pi} = S_2^{\pi\pi} = 1$ (i.e., neglecting both elastic $\pi\pi \rightleftharpoons \pi\pi$ and inelastic $\pi\pi \rightleftharpoons D\bar{D}$ rescattering effects), we arrive from Eqs. (2) and (3) at [‡]

$$
A(B_d^0 \to \pi^+\pi^-)_0 = \frac{G_F}{\sqrt{2}} \left[V_{ub}^* V_{ud} \left(a_1 + a_4^u + a_6^u \xi_\pi \right) + V_{cb}^* V_{cd} \left(a_4^c + a_6^c \xi_\pi \right) \right] T_\pi ,
$$

[‡]Note that there is a factor of $1/2$ for the phase space of $B_d^0 \to \pi^0\pi^0$ due to identical final-state particles. This factor has already been absorbed into the transition amplitude $A(B_d^0 \to \pi^0\pi^0)$ in Eqs. (2) and (6).

$$A(B_d^0 \to \pi^0\pi^0)_{\mathbf{0}} = \frac{G_F}{2} \left[V_{ub}^* V_{ud} \left(a_2 - a_4^u - a_6^u \xi_\pi \right) - V_{cb}^* V_{cd} \left(a_4^c + a_6^c \xi_\pi \right) \right] T_\pi ,$$

$$A(B_u^+ \to \pi^+\pi^0)_{\mathbf{0}} = \frac{G_F}{2} V_{ub}^* V_{ud} (a_1 + a_2) T_\pi , \tag{6}$$

where the subscript "**0**" denotes the absence of final-state interactions at the hadron level. Such a result has been presented in Refs. [4, 5]. The branching ratios of $B \to \pi\pi$ decays can then be computed, under isospin symmetry, by using the formula

$$\mathcal{B}(B \to \pi\pi)_{\mathbf{0}} = \frac{\tau_B \sqrt{m_B^2 - 4m_\pi^2}}{16\pi m_B^2} \left| A(B \to \pi\pi)_{\mathbf{0}} \right|^2 , \tag{7}$$

where τ_B denotes the B-meson lifetime. To obtain the quantitative results for $\mathcal{B}(B \to \pi\pi)_{\mathbf{0}}$, we input the following typical values of the quark mixing parameters: $|V_{ud}| = 0.9735$, $|V_{cd}| = 0.224$, $|V_{cb}| = 0.0402$, $|V_{ub}|/|V_{cb}| = 0.090$, and $\gamma \equiv \arg(-V_{ub}^* V_{ud} V_{cb} V_{cd}^*) = 65°$ [16]. Furthermore $\tau_B = 1.6$ ps, $f_\pi = 130.7$ MeV, and $F_0^{B\pi}(m_\pi^2) \approx F_0^{B\pi}(0) = 0.3$ are used [16, 17]. Following Beneke *et al.* in Ref. [4], we adopt $a_1 = 1.038 + 0.018i$, $a_2 = 0.082 - 0.080i$, $a_4^u = -0.029 - 0.015i$, and $a_4^c = -0.034 - 0.008i$ at the scale $\mu = m_b$, and neglect the formally power-suppressed QCD coefficients $a_6^{u,c}$ in the heavy quark limit. The predictions for the branching ratios of $B \to \pi\pi$ turn out to be [§]

$$\mathcal{B}(B_d^0 \to \pi^+\pi^-)_{\mathbf{0}} \approx 1.0 \times 10^{-5} ,$$
$$\mathcal{B}(B_d^0 \to \pi^0\pi^0)_{\mathbf{0}} \approx 1.4 \times 10^{-8} ,$$
$$\mathcal{B}(B_u^+ \to \pi^+\pi^0)_{\mathbf{0}} \approx 5.8 \times 10^{-6} ; \tag{8}$$

and

$$\mathcal{B}(\bar{B}_d^0 \to \pi^+\pi^-)_{\mathbf{0}} \approx 9.8 \times 10^{-6} ,$$
$$\mathcal{B}(\bar{B}_d^0 \to \pi^0\pi^0)_{\mathbf{0}} \approx 1.5 \times 10^{-7} ,$$
$$\mathcal{B}(B_u^- \to \pi^-\pi^0)_{\mathbf{0}} \approx 5.8 \times 10^{-6} . \tag{9}$$

First of all we observe that $\mathcal{B}(B_d^0 \to \pi^+\pi^-)_{\mathbf{0}}$ deviates almost a factor of 2 from the CLEO data given in Eq. (1), but it is in good agreement with the preliminary BABAR data. At present it remains too early to claim any discrepancy between the theoretical prediction and the experimental measurements. Secondly, the branching ratios of $B_d^0 \to \pi^0\pi^0$ and $\bar{B}_d^0 \to \pi^0\pi^0$ are rather sensitive to the values of the QCD coefficients and the weak phase γ. The implication of $\mathcal{B}(B_d^0 \to \pi^0\pi^0)_{\mathbf{0}}/\mathcal{B}(\bar{B}_d^0 \to \pi^0\pi^0)_{\mathbf{0}} \approx 0.1$ is quite clear: the direct CP-violating asymmetry between these two decay modes might be of $\mathcal{O}(1)$. In addition, $B_u^\pm \to \pi^\pm\pi^0$ transitions can be calculated in a relatively reliable way at the tree level, thus a measurement of their branching ratios will test the validity of the QCD-factorization approximation. Actually the preliminary CLEO data $\mathcal{B}(B_u^\pm \to \pi^\pm\pi^0) = (5.4^{+2.1}_{-2.0} \pm 1.5) \times 10^{-6}$ [1], though not yet formally announced, do agree very well with the theoretical prediction.

Now we recalculate the branching ratios of $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ decays by taking the final-state rescattering effects into account. We assume that the inelastic rescattering matrix

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[§] The upper bound on the branching ratio of $B_d^0 \to \pi^0\pi^0$ has been reported by the CLEO Collaboration [1]: $\mathcal{B}(B_d^0 \to \pi^0\pi^0) < 5.7 \times 10^{-6}$; and the upper bounds on the branching ratio of $B_u^\pm \to \pi^\pm\pi^0$ have been reported by both CLEO and BELLE Collaborations: $\mathcal{B}(B_u^\pm \to \pi^\pm\pi^0) < 1.27 \times 10^{-5}$ (CLEO [1]) and $\mathcal{B}(B_u^\pm \to \pi^\pm\pi^0) < 1.01 \times 10^{-5}$ (BELLE [2]).
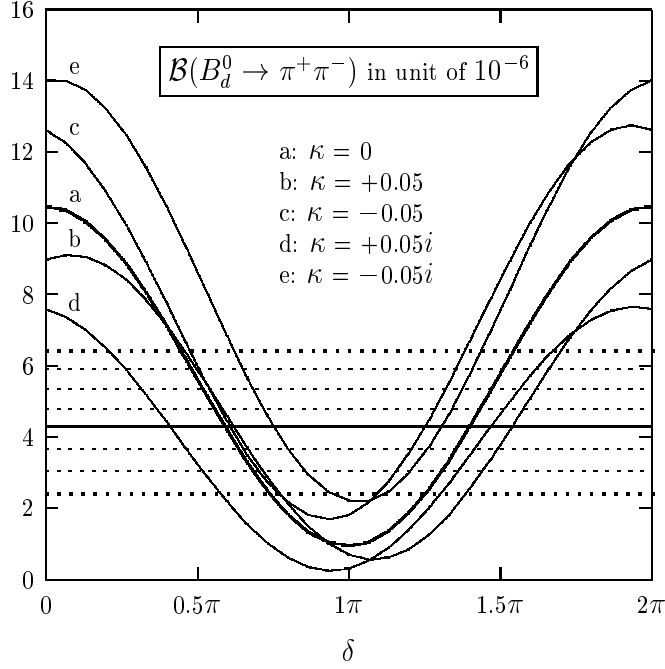
Figure 1: The branching ratio of $B_d^0 \to \pi^+\pi^-$ predicted in the factorization approximation with different input values of the inelastic rescattering parameter κ and the isospin phase shift δ. The dashed region is favored by the CLEO data [1].

S is approximately diagonal (i.e., $S_2^{\pi\pi} \approx S_0^{\pi\pi} \approx 1$) and its off-diagonal element $S_0^{\pi D}$ is only of $\mathcal{O}(10^{-2})$ in magnitude. In this reasonable assumption, which should not be far away from reality, one can simplify Eq. (2) and arrive at

$$A(B_d^0 \to \pi^+\pi^-) \approx \frac{G_{\rm F}}{\sqrt{2}} \left\{ V_{ub}^* V_{ud} \left[\frac{1}{3}(a_1 + a_2)e^{i\delta} + \frac{1}{3}(2a_1 - a_2) + (a_4^u + a_6^u \xi_\pi) \right] \right.$$
$$\left. + V_{cb}^* V_{cd} \left[\sqrt{2}\, a_1 \kappa + (a_4^c + a_6^c \xi_\pi) \right] \right\} T_\pi e^{i\delta_0} \; ,$$

$$A(B_d^0 \to \pi^0\pi^0) \approx \frac{G_{\rm F}}{2} \left\{ V_{ub}^* V_{ud} \left[\frac{2}{3}(a_1 + a_2)e^{i\delta} - \frac{1}{3}(2a_1 - a_2) - (a_4^u + a_6^u \xi_\pi) \right] \right.$$
$$\left. - V_{cb}^* V_{cd} \left[\sqrt{2}\, a_1 \kappa + (a_4^c + a_6^c \xi_\pi) \right] \right\} T_\pi e^{i\delta_0} \; , \tag{10}$$

where $\delta = \delta_2 - \delta_0$ and $\kappa = S_0^{\pi D} T_D / T_\pi$. In obtaining Eq. (10), we have neglected the small quantities of $\mathcal{O}(|a_4^{u,c}| \cdot |\kappa|)$ and $\mathcal{O}(|a_6^{u,c}| \cdot |\kappa|)$. Indeed we find $T_D/T_\pi \approx 3$ for $f_D = 200$ MeV and $F_0^{BD}(0) = 0.7$ [17], with the help of Eq. (5). Therefore $|\kappa| \sim \mathcal{O}(|S_0^{\pi D}|) \sim \mathcal{O}(10^{-2})$ holds. We observe that $A(B_d^0 \to \pi^+\pi^-)_{\bf 0}$ and $A(B_d^0 \to \pi^0\pi^0)_{\bf 0}$ in Eq. (6) can be reproduced from Eq. (10) by setting $\kappa = 0$ and $\delta = \delta_0 = 0$. The free parameters κ and δ are likely to affect the branching ratios of $B_d \to \pi\pi$ transitions significantly. For illustration we take $\kappa = -0.05, -0.05i, 0, +0.05i, +0.05$, respectively, to compute $\mathcal{B}(B_d \to \pi\pi)$ as a function of $\delta \in [0, 2\pi]$ by using Eq. (7). The numerical results are shown in Figs. 1 and 2.

Fig. 1 indicates that in the absence of inelastic $\pi\pi \rightleftharpoons D\bar{D}$ rescattering a good agreement between the theoretical value of $\mathcal{B}(B_d^0 \to \pi^+\pi^-)$ and the CLEO data invokes $\delta \sim 0.5\pi$ or 1.5π; i.e., there may exist significant elastic $\pi\pi \rightleftharpoons \pi\pi$ rescattering. This result is apparently consistent with the analyses made in Refs. [8, 9]. Taken the final-state $\pi\pi \rightleftharpoons D\bar{D}$ rescattering
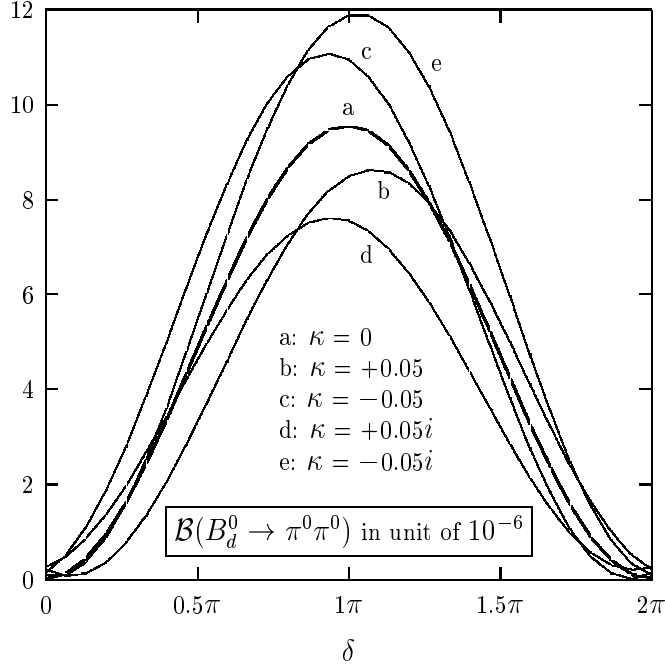
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Figure 2: The branching ratio of $B_d^0 \to \pi^0\pi^0$ predicted in the factorization approximation with different input values of the inelastic rescattering parameter κ and the isospin phase shift δ.

effect into account, the naive value of $\mathcal{B}(B_d^0 \to \pi^+\pi^-)_0$ given before can be lowered even in the case $\delta = 0$: e.g., we obtain $\mathcal{B}(B_d^0 \to \pi^+\pi^-)/\mathcal{B}(B_d^0 \to \pi^+\pi^-)_0 \approx 0.7$ by taking $\mathrm{Re}\kappa = 0$ and $\mathrm{Im}\kappa = 0.05$. We therefore conclude that both kinds of final-state interactions are important and non-negligible. Indeed there is a rather large (κ, δ)-parameter space, as shown in Fig. 1, in which the present data of CLEO, BELLE or BABAR on $B_d^0 \to \pi^+\pi^-$ and $\bar{B}_d^0 \to \pi^+\pi^-$ decays can well be accommodated.

Fig. 2 shows our prediction for $\mathcal{B}(B_d^0 \to \pi^0\pi^0)$ in the presence of both $\pi\pi \rightleftharpoons \pi\pi$ and $\pi\pi \rightleftharpoons D\bar{D}$ rescattering effects. We observe that the naive value of $\mathcal{B}(B_d^0 \to \pi^0\pi^0)_0$ can be enhanced up to two orders of magnitude. In the region of $\delta \sim 0.5\pi$ or 1.5π, $\mathcal{B}(B_d^0 \to \pi^0\pi^0) \sim 2 \times 10^{-6}$ to 8×10^{-6} is expected for different values of κ. Considering the CLEO upper limit $\mathcal{B}(B_d^0 \to \pi^0\pi^0) < 5.7 \times 10^{-6}$ [1], however, we see that the region $\delta < 0.5\pi$ or $\delta > 1.5\pi$ is more favored [¶]. An experimental determination of the branching ratios of $B_d^0 \to \pi^0\pi^0$ and $\bar{B}_d^0 \to \pi^0\pi^0$ transitions at the level of 10^{-6} should signify significant final-state interactions.

We proceed to calculate the direct CP-violating asymmetries in B_d^0 vs $\bar{B}_d^0 \to \pi^+\pi^-$ and $\pi^0\pi^0$ decays, defined respectively as

$$\mathcal{A}(\pi^+\pi^-) = \frac{|A(B_d^0 \to \pi^+\pi^-)|^2 - |A(\bar{B}_d^0 \to \pi^+\pi^-)|^2}{|A(B_d^0 \to \pi^+\pi^-)|^2 + |A(\bar{B}_d^0 \to \pi^+\pi^-)|^2} \, ,$$

$$\mathcal{A}(\pi^0\pi^0) = \frac{|A(B_d^0 \to \pi^0\pi^0)|^2 - |A(\bar{B}_d^0 \to \pi^0\pi^0)|^2}{|A(B_d^0 \to \pi^0\pi^0)|^2 + |A(\bar{B}_d^0 \to \pi^0\pi^0)|^2} \, . \qquad (11)$$

These asymmetries can be observed time-independently on the $\Upsilon(4S)$ resonance with a trivial dilution factor $1/(1+x_d^2) \approx 0.66$ due to B_d^0-$\bar{B}_d^0$ mixing [19], or time-dependently at asymmetric

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[¶]Note that $\delta \approx 11°$ is expected in the Regge model [18].

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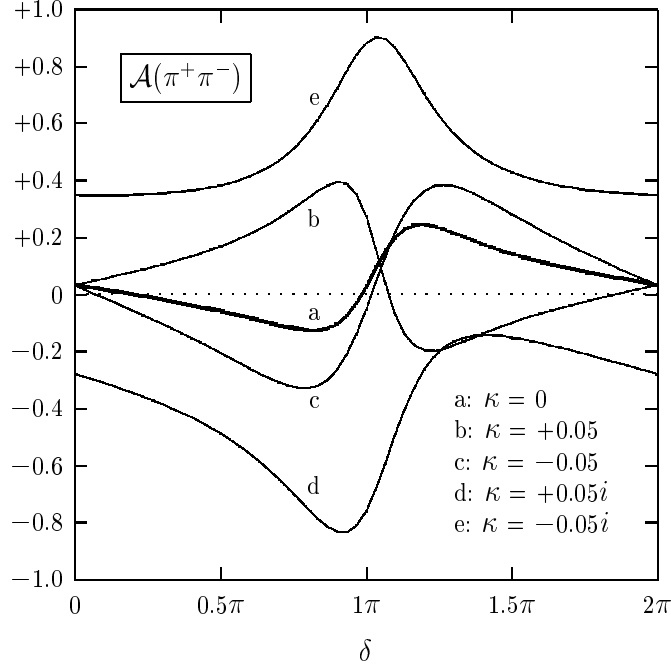
Figure 3: The CP-violating asymmetry of B_d^0 vs $\bar{B}_d^0 \to \pi^+\pi^-$ decays predicted in the factorization approximation with different input values of the inelastic rescattering parameter κ and the isospin phase shift δ.

B-meson factories running around the $\Upsilon(4S)$ energy threshold [20]. The numerical results of $\mathcal{A}(\pi^+\pi^-)$ and $\mathcal{A}(\pi^0\pi^0)$ are shown in Figs. 3 and 4, where we have used the same values as before for the relevant input parameters. Some comments are in order:

(a) In the absence of final-state interactions at the hadron level (i.e., $\delta = 0$ and $\kappa = 0$), the CP asymmetries $\mathcal{A}(\pi^+\pi^-) \approx 3\%$ and $\mathcal{A}(\pi^0\pi^0) \approx -82\%$ are a consequence of the interference between tree-level and penguin amplitudes, where the non-trivial strong phase shift arises from the penguin quark-loop function [21]. Note that we have neglected possible effects from the electroweak penguins [22], the space-like penguins [23], and the self-interference of different penguin loops, as they are generally expected to be insignificant in the transitions under consideration.

(b) If only the $\pi\pi \rightleftharpoons \pi\pi$ rescattering effect is "switched on", $\mathcal{A}(\pi^+\pi^-)$ undergoes an oscillation with increasing values of δ and its magnitude can be as large as 25% for $\delta \approx 1.2\pi$, while the magnitude of $\mathcal{A}(\pi^0\pi^0)$ always decreases when δ deviates from 0 or 2π. In this case CP violation remains resulting from the interference between tree-level and penguin amplitudes, but the relevant isospin phase differences may play a more important role than the strong phase shifts induced by penguin loops at the quark level.

(c) If only the $\pi\pi \rightleftharpoons D\bar{D}$ rescattering effect is "switched on", the magnitude of $\mathcal{A}(\pi^+\pi^-)$ can remarkably be enhanced (e.g., $\mathcal{A}(\pi^+\pi^-) \approx 35\%$ for $\kappa = -0.05i$), but that of $\mathcal{A}(\pi^0\pi^0)$ becomes smaller than in the case $\kappa = 0$. There are two sources of CP violation: one is the interference between tree-level and penguin amplitudes, and the other is the interference between two different tree-level amplitudes as a result of $\pi\pi \rightleftharpoons D\bar{D}$ rescattering. The latter is measured by κ, whose magnitude and phase can both affect the CP-violating asymmetries in a significant way.

(d) In general $\pi\pi \rightleftharpoons \pi\pi$ and $\pi\pi \rightleftharpoons D\bar{D}$ rescattering effects should both be taken into account.
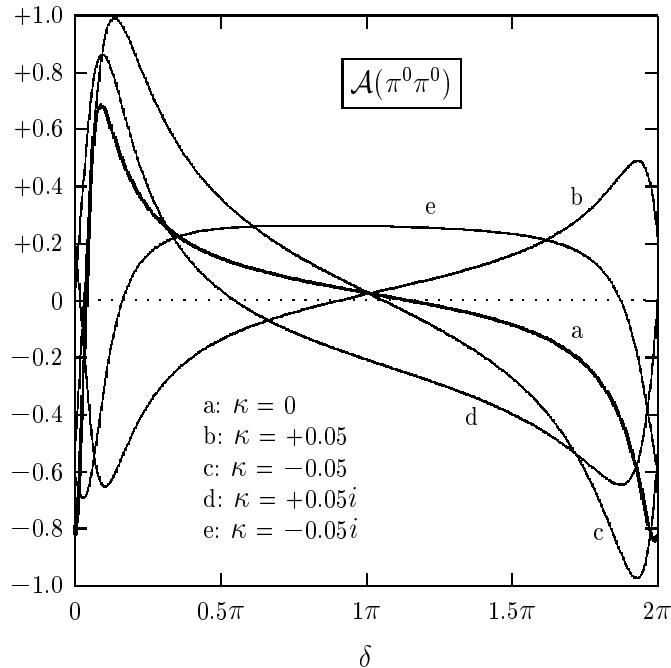
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Figure 4: The CP-violating asymmetry of B_d^0 vs $\bar{B}_d^0 \to \pi^0 \pi^0$ decays predicted in the factorization approximation with different input values of the inelastic rescattering parameter κ and the isospin phase shift δ.

It is then possible to have $|\mathcal{A}(\pi^+ \pi^-)| \sim \mathcal{O}(1)$ and $|\mathcal{A}(\pi^0 \pi^0)| \sim \mathcal{O}(1)$ for appropriate values of the input parameters. While large CP asymmetries are likely to be measured in $B_d \to \pi\pi$ decays, to pin down the true mechanism of CP violation will be very difficult. Furthermore, the indirect CP-violating signals in such decay modes (arising from the interplay of direct decay and B_d^0-$\bar{B}_d^0$ mixing) are unavoidably contaminated by significant final-state rescattering effects [13, 24]. It is therefore a big challenge to extract any information on the weak CP-violating phases from $B_d \to \pi\pi$ transitions.

In summary, we have demonstrated the significance of elastic and inelastic final-state rescattering effects in $B_d \to \pi\pi$ decays. Our treatment of the complicated inelastic final-state interactions is soley to take the simple $\pi\pi \rightleftharpoons D\bar{D}$ rescattering into account, hence it remains quite preliminary and can only serve for illustration. Nevertheless, the present experimental data on $B \to \pi\pi$ decays can well be understood in our approach without fine-tuning of the input parameters, and large CP-violating asymmetries are expected to manifest themselves in such charmless rare processes. We remark that further effort is desirable towards a deeper understanding of the dynamics of nonleptonic B decays.

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